Aufgaben zu partiellen Ableitungen

Aufgabe
Ermitteln Sie die unbestimmten Integrale mit Hilfe der Methode der partiellen Integration.

1) $\displaystyle\int x \cdot e^x dx$

2) $\displaystyle\int \ln x dx$

3) $\displaystyle\int \ln^2 x dx$

4) $\displaystyle\int x^2 \sin x dx$

5) $\displaystyle\int \sin^2 x dx$

6) $\displaystyle\int e^x \sin x dx$

7) $\displaystyle\int \frac{\mathrm{arsinh} x}{x^2} dx$

8) $\displaystyle\int x \ln x dx$

9) $\displaystyle\int \frac{\ln x}{x} dx$

Lösungen

1)

$$\int x\cdot e^x dx = x\cdot\int e^x - \int\left(1\cdot\int(e^x dx)\right)dx = x\cdot e^x - \int e^x dx = x\cdot e^x - e^x + c = e^x(x-1)+c$$

2)

$$\int \ln x dx = \int(\ln x\cdot 1)dx = \ln x\cdot x - \int\left(\frac{1}{x}\cdot x\right)dx = x\ln x - \int 1dx = x\ln x - x + c$$

3)

$$\int \ln^2 x{\cdot}xdx = \int \ln x{\cdot}\ln xdx = \ln x(x\ln x - x) - \int\frac{1}{x}(x\ln x - x)dx = x\ln^2 x - x\ln x - \int(\ln x - 1)dx =$$

$$= x\ln^2 x - x\ln x - (x\ln x - x) + x + c = x\ln^2 x - x\ln x - x\ln x + x + x + c = x\ln^2 x - 2x\ln x + 2x + c$$

4)

$$\int x^2\sin xdx = x^2(-\cos x) - \int 2x\cdot(-\cos x)dx = -x^2\cos x + 2\int -x\cos xdx =$$

$$= -x^2\cos x + 2(x\sin x - \int 1\cdot\sin xdx) = -x^2\cos x + 2(x\sin x + \cos x) + c =$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + c$$

5)

$$\int \sin^2 xdx = \int \sin^2 x\cdot 1dx = \sin^2 x\cdot x - \int 2\sin x\cos x\cdot xdx = x\sin^2 x - \int x\sin 2xdx =$$

$$= x\sin^2 x - (x\int\sin 2xdx - \int 1\cdot(\int\sin 2xdx)dx) = x\sin^2 x - x\frac{-\cos 2x}{2} + \int\frac{-cos2x}{2}dx =$$

$$= x\sin^2 x + \frac{x\cos 2x}{2} - \frac{1}{2}\int\cos 2xdx = x\sin^2 x + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4} + c =$$

$$= \frac{2x\sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c = \frac{x\cos^2 x}{2} + \frac{x\sin^2 x}{2} - \frac{\sin 2x}{4} + c =$$

$$= \frac{x}{2}(\cos^2 x + \sin^2 x) - \frac{\sin 2x}{4} + c = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

6)

$$\int e^x\sin xdx = e^x\sin x - \int e^x\cos xdx = e^x\sin x - e^x\cos x - \int e^x\sin xdx$$

$$\int e^x\sin xdx = \frac{1}{2}e^x(\sin x - \cos x) + c$$

7)

$$\int\frac{1}{x^2}\mathrm{arsinh}xdx = -\frac{1}{x}\mathrm{arsinh}x + \frac{dx}{x\sqrt{1+x^2}} = -\frac{1}{x}\mathrm{arsinh}x + \int\frac{dz}{z^2-1} = -\frac{1}{x}\mathrm{arsinh}x - \mathrm{arcoth}\sqrt{x^2+1} + c$$

8)

$$\int x\ln xdx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$$

9)

$$\int\frac{\ln x}{x}dx = \frac{1}{2}\ln^2 x + c$$