Aufgaben zur partiellen Integration

Ermitteln Sie die unbestimmten Integrale mit Hilfe der Methode der partiellen Integration.

1) $\int x \cdot e^x \, dx$

2) $\int \ln x \, dx$

3) $\int \ln^2 x \, dx$

4) $\int x^2 \sin x \, dx$

5) $\int \sin^2 x \, dx$

6) $\int x^2 \sin x \, dx$

7) $\int e^x \sin x \, dx$

8) $\int 1/x^2 \operatorname{arsinh} x \, dx$

9) $\int x \ln x \, dx$

10) $\int \ln x / x \;\; dx$

Lösungen

$\int x \cdot e^x\, dx = x \cdot \int e^x\, dx - \int 1 \cdot (\int e^x\, dx)\, dx = x \cdot e^x - \int e^x\, dx = x \cdot e^x - e^x + c = e^x (x - 1) + c$

$\int \ln x\, dx = \int \ln x \cdot 1\, dx = \ln x \cdot x - \int 1/x \cdot x\, dx = x \ln x - \int 1\, dx = x \ln x - x + c$

$\int \ln^2 x\, dx = \int \ln x \cdot \ln x\, dx = \ln x\, (x \ln x - x) - \int 1/x\, (x \ln x - x)\, dx = x \ln^2 x - x \ln x - \int (\ln x - 1)\, dx =$

$= x \ln^2 x - x \ln x - (x \ln x - x) + x + c = x \ln^2 x - x \ln x - x \ln x + x + x + c = x \ln^2 x - 2x \ln x + 2x + c$

$\int x^2 \sin x\, dx = x^2 (-\cos x) - \int 2x (-\cos x)\, dx = -x^2 \cos x + 2 \int -x \cos x\, dx = -x^2 \cos x + 2\, [x \sin x - \int 1 \sin x\, dx] =$

$= -x^2 \cos x + 2\, [x \sin x + \cos x] + c = -x^2 \cos x + 2x \sin x + 2 \cos x + c$

$\int \sin^2 x\, dx = \int \sin^2 x \cdot 1\, dx = \sin^2 x \cdot x - \int 2 \sin x \cos x \cdot x\, dx = x \sin^2 x - \int x \sin 2x\, dx =$

$= x \sin^2 x - [x \int \sin 2x\, dx - \int 1 (\int \sin 2x\, dx)\, dx] = x \sin^2 x - x ([- \cos 2x]/2) + \int (- [\cos 2x]/2)\, dx =$

$= x \sin^2 x + [x \cos 2x]/2 - ½ \int \cos 2x\, dx = x \sin^2 x + [x \cos 2x]/2 - [\sin 2x]/4 + c =$

$= [2x \sin^2 x]/2 + [x (\cos^2 x - \sin^2 x)]/2 - [\sin 2x]/4 + c = [x \cos^2 x]/2 + [x \sin^2 x]/2 - [\sin 2x]/4 + c =$

$= x/2\, (\cos^2 x + \sin^2 x) - [\sin 2x]/4 + c = x/2 - [\sin 2x]/4 + c$

$\int x^2 \sin x\, dx = -x^2 \cos x + \int 2x \cos x\, dx = -x^2 \cos x + 2x \sin x - \int 2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$

$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx = e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$ ... $\int e^x \sin x\, dx = ½\, e^x (\sin x - \cos x) + C$

$\int 1/x^2\, \text{arsinh}\, x\, dx = -1/x\, \text{arsinh}\, x + \int dx/(x \sqrt{(1+x^2)}) = - 1/x\, \text{arsinh}\, x + \int dz/(z^2-1) = -1/x\, \text{arsinh}\, x - \text{arcoth} \sqrt{(x^2+1)} + C$

$\int x \ln x\, dx = 1/2\, x^2 \ln x - 1/4\, x^2 + C$

$\int \ln x / x\, dx = 1/2 \ln^2 x + C$