# 1st Vietnam 1962 problems

**Problem**
Prove that $1/(1/a + 1/b) + 1/(1/c + 1/d) \le 1/(1/(a+c) + 1/(b+d))$ for positive reals a, b, c, d.

**Solution**
A straightforward, if inelegant, approach is to multiply out and expand everything. All terms cancel except four and we are left with $2abcd \le a^2d^2 + b^2c^2$, which is obviously true since $(ad - bc)^2 \ge 0$.

**Problem 2**
$f(x) = (1 + x)(2 + x^2)^{1/2}(3 + x^3)^{1/3}$. Find $f'(-1)$.

**Solution**
Differentiating gives $f'(x) = (2 + x^2)^{1/2}(3 + x^3)^{1/3}$ + terms with factor $(1 + x)$. Hence $f'(-1) = 3^{1/2}2^{1/3}$.

**Problem 3**
ABCD is a tetrahedron. A' is the foot of the perpendicular from A to the opposite face, and B' is the foot of the perpendicular from B to the opposite face. Show that AA' and BB' intersect iff AB is perpendicular to CD. Do they intersect if AC = AD = BC = BD?

**Solution**
Let the ray AB' meet CD at X and the ray BA' meet CD at Y. If AB' and A'B intersect, then X = Y. Let L be the line through A' parallel to CD. Then L is perpendicular to AA'. Hence CD is perpendicular to AA'. Similarly, let L' be the line through B' parallel to CD. Then L' is perpendicular to BB', and hence CD is perpendicular to BB'. So CD is perpendicular to two non-parallel lines in the plane ABX. Hence it is perpendicular to all lines in the plane ABX and, in particular, to AB.

Suppose conversely that AB is perpendicular to CD. Consider the plane ABY. CD is perpendicular to AB and to AA', so CD is perpendicular to the plane. Similarly CD is perpendicular to the plane ABX. But it can only be perpendicular to a single plane through AB. Hence X = Y and so AA' and BB' belong to the same plane and therefore meet.

**Problem 4**
The tetrahedron ABCD has BCD equilateral and AB = AC = AD. The height is h and the angle between ABC and BCD is $\alpha$. The point X is taken on AB such that the plane XCD is perpendicular to AB. Find the volume of the tetrahedron XBCD.

**Problem 5**
Solve the equation $\sin^6 x + \cos^6 x = 1/4$.

**Solution**
Put $a = \sin^2 x$, $b = \cos^2 x$. Then a and b are non-negative with sum 1, so we may put $a = 1/2 + h$, $b = 1/2 - h$. Then $a^3 + b^3 = 1/4 + 3h^2 \ge 1/4$ with equality iff $h = 0$. Hence x is a solution of the equation given iff $\sin^2 x = \cos^2 x = 1/2$ or x is an odd multiple of $\pi/4$.

## 2nd VMO 1963

**Problem 1**
A conference has 47 people attending. One woman knows 16 of the men who are attending, another knows 17, and so on up to the last woman who knows all the men who are attending. Find the number of men and women attending the conference.

**Solution**
Suppose there are m women. Then the last woman knows 15+m men, so 15+2m = 47, so m = 16. Hence there are 31 men and 16 women.

**Problem 2**
For what values of m does the equation $x^2 + (2m + 6)x + 4m + 12 = 0$ has two real roots, both of them greater than -2.

**Answer**
$m \le -3$

**Solution**
For real roots we must have $(m+3)^2 \ge 4m+12$ or $(m-1)(m+3) \ge 0$, so $m \ge 1$ or $m \le -3$. If $m \ge 1$, then $-(2m+6) \le -8$, so at least one of the roots is $< -2$. So we must have $m \le -3$.

The roots are $-(m+3) \pm\sqrt{(m^2+2m-3)}$. Now $-(m+3) \ge 0$, so $-(m+3) + \sqrt{(m^2+2m-3)} \ge 0 > -2$. So we need $-(m+3) - \sqrt{(m^2+2m-3)} > -2$, or $\sqrt{(m^2+2m-3)} < -m-1 = \sqrt{(m^2+2m+1)}$, which is always true.

**Problem 3**
Solve the equation $\sin^3 x \cos 3x + \cos^3 x \sin 3x = 3/8$.

**Answer**
$7½^o + k90^o$ or $37½^o + k90^o$

**Solution**
We have $\sin 3x = 3 \sin x - 4 \sin^3 x$, $\cos 3x = 4 \cos^3 x - 3 \cos x$. So we need $4 \sin^3 x \cos^3 x - 3 \sin^3 x \cos x + 3 \sin x \cos^3 x - 4 \sin^3 x \cos^3 x = 3/8$ or $8 \sin x \cos x(\cos^2 x - \sin^2 x) = 1$, or $4 \sin 2x \cos 2x = 1$ or $\sin 4x = 1/2$. Hence $4x = 30^o + k360^o$ or $150^o + k360^o$. So $x = 7½^o + k90^o$ or $37½^o + k90^o$.

**Problem 4**
The tetrahedron SABC has the faces SBC and ABC perpendicular. The three angles at S are all $60^o$ and SB = SC = 1. Find its volume.

**Problem 5**
The triangle ABC has perimeter p. Find the side length AB and the area S in terms of $\angle A$, $\angle B$ and p. In particular, find S if p = 23.6, A = 52.7 deg, B = 46 4/15 deg.

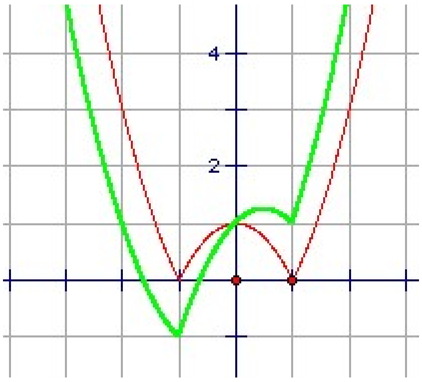
## 3rd VMO 1964

**Problem A1**
Find cos x + cos(x + 2π/3) + cos(x + 4π/3) and sin x + sin(x + 2π/3) + sin(x + 4π/3).

**Solution**
Using cos(A+B) = cos A cos B - sin A sin B, we have cos(x + 2π/3) = -(1/2) cos x + (√3)/2 sin x, cos(x + 4π/3) = -(1/2) cos x - (√3)/2 sin x. Hence cos x + cos(x + 2π/3) + cos(x + 4π/3) = 0. Similarly, sin(x + 2π/3) = -1/2 sin x + (√3)/2 cos x, sin(x + 4π/3) = -1/2 sin x - (√3)/2 cos x, so sin x + sin(x + 2π/3) + sin(x + 4π/3) = 0.

**Problem A2**
Draw the graph of the functions $y = | x^2 - 1 |$ and $y = x + | x^2 - 1 |$. Find the number of roots of the equation $x + | x^2 - 1 | = k$, where k is a real constant.

**Answer**
0 for k < -1 , 1 for k = -1 , 2 for -1 < k < 1 , 3 for k = 1 , 4 for 1 < k < 5/4 , 3 for k = 5/4 , 2 for k > 5/4




**Solution**

It is clear from the graph that there are no roots for k < -1, and one root for k = -1 (namely x = -1). Then for k > -1 there are two roots except for a small interval [1, 1+h]. At k = 1, there are 3 roots (x = -2, 0, 1). The upper bound is at the local maximum between 0 and 1. For such x, y = x + 1 - x2 = 5/4 - (x - 1/2)2, so the local maximum is at 5/4. Thus there are 3 roots at k = 5/4 and 4 roots for k ∈ (1, 5/4).

**Problem A3**
Let O be a point not in the plane p and A a point in p. For each line in p through A, let H be the foot of the perpendicular from O to the line. Find the locus of H.

**Answer:** circle diameter AB, where OB is the normal to p

**Solution**
Let B be the foot of the perpendicular from O to p. We claim that the locus is the circle diameter AB. Any line in p through A meets this circle at one other point K (except for the tangent to the circle at A, but in that case A is obviously the foot of the perpendicular from O to the line). Now BK is perpendicular to AK, so OK is also perpendicular to AK, and hence K must be the foot of the perpendicular from O to the line.

**Problem A4**
Define the sequence of positive integers $f_n$ by $f_0 = 1$, $f_1 = 1$, $f_{n+2} = f_{n+1} + f_n$. Show that $f_n = (a^{n+1} - b^{n+1})/\sqrt{5}$, where a, b are real numbers such that a + b = 1, ab = -1 and a > b.

**Solution**
Put a = (1+√5)/2, b = (1-√5)/2. Then a, b are the roots of $x^2 - x - 1 = 0$ and satisfy a + b = 1, ab = -1. We show by induction that $f_n = (a^{n+1} - b^{n+1})/\sqrt{5}$. We have $f_0 = (a-b)/\sqrt{5} = 1$, $f_1 = (a^2-b^2)/\sqrt{5} = (a+1 - b-1)/\sqrt{5} = 1$, so the result is true for n = 0, 1. Finally, suppose $f_n = (a^{n+1} - b^{n+1})/\sqrt{5}$ and $f_{n+1} = (a^{n+2} - b^{n+2})/\sqrt{5}$. Then $f_{n+2} = f_{n+1} + f_n = (1/\sqrt{5})(a^{n+1}(a+1) - b^{n+1}(b+1)\ ) = (a^{n+1}a^2 - b^{n+1}b^2)/\sqrt{5}$, so the result is true for n+1.

# 4th Vietnam 1965 problems

**Problem 1**
At time $t = 0$, a lion L is standing at point O and a horse H is at point A running with speed v perpendicular to OA. The speed and direction of the horse does not change. The lion's strategy is to run with constant speed u at an angle $0 < \varphi < \pi/2$ to the line LH. What is the condition on u and v for this strategy to result in the lion catching the horse? If the lion does not catch the horse, how close does he get? What is the choice of $\varphi$ required to minimise this distance?

**Problem 2**
AB and CD are two fixed parallel chords of the circle S. M is a variable point on the circle. Q is the intersection of the lines MD and AB. X is the circumcenter of the triangle MCQ. Find the locus of X. What happens to X as M tends to (1) D, (2) C? Find a point E outside the plane of S such that the circumcenter of the tetrahedron MCQE has the same locus as X.

**Problem 3**
m an n are fixed positive integers and k is a fixed positive real. Show that the minimum value of $x_1^m + x_2^m + x_3^m + \dots + x_n^m$ for real $x_i$ satisfying $x_1 + x_2 + \dots + x_n = k$ occurs at $x_1 = x_2 = \dots = x_n$.

# 5th Vietnam 1966 problems

**Problem 1**
[missing]

**Problem 2**
a, b are two fixed lines through O. Variable lines x, y are parallel. x intersects a at A and b at C, y intersects a at B and b at D. The lines AD and BC meet at M. The line through M parallel to x meets a at L and b at N. What can you say about L, M, N? Find the locus M.

**Problem 3**
(1) ABCD is a rhombus. A tangent to the inscribed circle meets AB, DA, BC, CD at M, N, P, Q respectively. Find a relationship between BM and DN.
(2) ABCD is a rhombus and P a point inside. The circles through P with centers A, B, C, D meet the four sides AB, BC, CD, DA in eight points. Find a property of the resulting octagon. Use it to construct a regular octagon.
(3) [unclear].

# 6th Vietnam 1967 problems

**Problem 1**
Draw the graph of the function $y = | x^3 - x^2 - 2x | /3 - | x + 1 |$.

**Problem 2**
A river flows at speed u. A boat has speed v relative to the water. If its velocity is at an angle $\alpha$ relative the direction of the river, what is its speed relative to the river bank? What $\alpha$ minimises the time taken to cross the river?

**Problem 3**
(1) ABCD is a rhombus. A tangent to the inscribed circle meets AB, DA, BC, CD at M, N, P, Q respectively. Find a relationship between BM and DN.
(2) ABCD is a rhombus and P a point inside. The circles through P with centers A, B, C, D meet the four sides AB, BC, CD, DA in eight points. Find a property of the resulting octagon. Use it to construct an equiangular octagon.
(3) Rotate the figure about the line AC to form a solid. State a similar result.

## 7th Vietnam 1968 problems

**Problem 1**
The real numbers a and b satisfy $a \geq b > 0$, $a + b = 1$. Show that $a^m - a^n \geq b^m - b^n > 0$ for any positive integers $m < n$. Show that the quadratic $x^2 - b^n x - a^n$ has two real roots in the interval $(-1, 1)$.

**Problem 2**
L and M are two parallel lines a distance d apart. Given r and x, construct a triangle ABC, with A on L, and B and C on M, such that the inradius is r, and angle A = x. Calculate angles B and C in terms of d, r and x. If the incircle touches the side BC at D, find a relation between BD and DC.

## 8th VMO 1969

**Problem A1**
A graph G has n + k points. A is a subset of n points and B is the subset of the other k points. Each point of A is joined to at least k - m points of B where $nm < k$. Show that there is a point in B which is joined every point in A.

**Solution**
This is just the pigeonhole principle. There are at least $n(k-m) > (n-1)k$ edges from points of A to points of B and there are k points in B. So some point of B must have at least n edges (otherwise there would be $\leq (n-1)k$ edges). That point is joined to every point of A.

**Problem A2**
Find all real x such that $0 < x < \pi$ and $8/(3 \sin x - \sin 3x) + 3 \sin^2 x \leq 5$.

**Answer:** $\pi/2$

**Solution**

We have $3 \sin x - \sin 3x = 4 \sin^3 x$. Put $s = \sin x$. Then we want $2/s^3 + 3s^2 \leq 5$. Note that since $0 < x < \pi$ we have s positive. But by AM/GM we have $1/s^3 + 1/s^3 + s^2 + s^2 + s^2 > 5$ with equality iff $s = 1$, so we must have $\sin x = 1$ and hence $x = \pi/2$.

**Problem A3**
The real numbers $x_1$, $x_4$, $y_1$, $y_2$ are positive and the real numbers $x_2$, $x_3$, $y_3$, $y_4$ are negative. We have $(x_i - a)^2 + (y_i - b)^2 \leq c^2$ for i = 1, 2, 3, 4. Show that $a^2 + b^2 \leq c^2$. State the result in geometric language.

**Solution**
Stated geometrically, the result is: if a disk includes a point in each quadrant, then it must also include the origin. We use the fact that a disk is convex. Let $P_i$ be the point $(x_i,y_i)$. The segment $P_1P_2$ must intersect the positive x-axis. By convexity, the point of intersection, call it X, must lie in the disk. Similarly, $P_3P_4$ must intersect the negative x-axis at some point Y, which must be in the disk. Then all points of the segment XY are in the disk and hence, in particular, the origin.

**Problem 4**
Two circles centers O and O', radii R and R', meet at two points. A variable line L meets the circles at A, C, B, D in that order and AC/AD = CB/BD. The perpendiculars from O and O' to L have feet H and H'. Find the locus of H and H'. If $OO'^2 < R^2 + R'^2$, find a point P on L such that PO + PO' has the smallest possible value. Show that this value does not depend on the position of L. Comment on the case $OO'^2 > R^2 + R'^2$.

## 9th VMO 1970

**Problem A1**
ABC is a triangle. Show that $\sin A/2 \sin B/2 \sin C/2 < 1/4$.

**Solution**
Put $x = A/2$, $y = B/2$. We have $\sin C/2 = \sin(90^o-x-y) = \cos(x+y)$. So we need to show that $\sin x \sin y \cos(x+y) < 1/4$, or $(\cos(x-y) - \cos(x+y))\cos(x+y) < 1/2$, or $2 \cos(x-y) \cos(x+y) < 1 + 2 \cos^2(x+y)$. But $2 \cos(x-y) \cos(x+y) \le \cos^2(x+y) + \cos^2(x-y) \le 1 + \cos^2(x+y) < 1 + 2 \cos^2(x+y)$ (since $0 < x,y < 90^o$.

**Problem A2**
Find all positive integers which divide $1890\cdot 1930\cdot 1970$ and are not divisible by 45.

**Answer**
$k\cdot 2^a 7^b 193^c 197^d$, where $k = 1, 3, 3^2, 3^3, 5, 3\cdot 5$, $a = 0, 1, 2,$ or $3$, $b = 0$ or $1$, $c = 0$ or $1$, $d = 0$ or $1$ (192 solutions in all)

**Solution**
$1890 = 2\cdot 3^3 5\cdot 7$, $1930 = 2\cdot 5\cdot 193$, $1970 = 2\cdot 5\cdot 197$ (and 193 and 197 are prime). So $1890\cdot 1930\cdot 1970 = 2^3 3^3 5^3 7\cdot 193\cdot 197$.

**Problem A3.**
The function $f(x, y)$ is defined for all real numbers x, y. It satisfies $f(x,0) = ax$ (where a is a non-zero constant) and if (c, d) and (h, k) are distinct points such that $f(c, d) = f(h, k)$, then $f(x, y)$ is constant on the line through (c, d) and (h, k). Show that for any real b, the set of points such that $f(x, y) = b$ is a straight line and that all such lines are parallel. Show that $f(x, y) = ax + by$, for some constant b.

**Problem B1**
AB and CD are perpendicular diameters of a circle. L is the tangent to the circle at A. M is a variable point on the minor arc AC. The ray BM, DM meet the line L at P and Q respectively. Show that $AP\cdot AQ = AB\cdot PQ$. Show how to construct the point M which gives BQ parallel to DP. If the lines OP and BQ meet at N find the locus of N. The lines BP and BQ meet the tangent at D at P' and Q' respectively. Find the relation between P' and Q'. The lines DP and DQ meet the line BC at P" and Q" respectively. Find the relation between P" and Q".

**Problem B2**
A plane p passes through a vertex of a cube so that the three edges at the vertex make equal angles with p. Find the cosine of this angle. Find the positions of the feet of the perpendiculars from the vertices of the cube onto p. There are 28 lines through two vertices of the cube and 20 planes through three vertices of the cube. Find some relationship between these lines and planes and the plane p.

# 10th Vietnam 1971 problems

**Problem A1**
m, n, r, s are positive integers such that: (1) $m < n$ and $r < s$; (2) m and n are relatively prime, and r and s are relatively prime; and (3) $\tan^{-1}m/n + \tan^{-1}r/s = \pi/4$. Given m and n, find r and s. Given n and s, find m and r. Given m and s, find n and r.

**Solution**
Put $k = \tan^{-1}m/n$, $h = \tan^{-1}r/s$. Then $h + k = \pi/4$, so $1 = \tan(h+k) = (\tan h + \tan k)/(1 - \tan h \tan k) = (r/s + m/n)/(1 - mr/ns) = (nr + ms)/(ns - mr)$. Hence $(m+n)(r+s) = 2ns$.

Suppose we are given m and n. We have $(m+n)r = (n-m)s$. If m and n have opposite parity, then m+n and m-n are coprime, so $r = n-m$, $s = m+n$. If m and n have the same parity, then m+n and m-n are both even, so $r = (n-m)/2$, $s = (m+n)/2$.

Suppose we are given n and s. wlog $n \ge s$. If $d > 1$ divides n and d divides m+n, then d divides m and n. Contradiction. So n must divide r+s. But $r < s \le n$, so $r+s < 2n$. Hence $r+s = n$, so $m+n = 2s$. Hence n and s must be relatively prime and $n < 2s$.

Suppose we are given m and s. Then $(m+n)(s-r) = 2ms$. If $d > 1$ divides m and m+n, then d divides m and n. Contradiction. So m must divide s-r. So if $d > 1$ divides m and s, then d divides r, so r and s are not coprime. Contradiction. Hence m and s must be coprime. Also m must be < s. But now we can take $r = s-m$, $n = 2s-m$.

**Problem B1**
ABCDA'B'C'D' is a cube (with ABCD and A'B'C'D' faces, and AA', BB', CC', DD' edges). L is a line which intersects or is parallel to the lines AA', BC and DB'. L meets the line BC at M (which may be the point at infinity). Let $m = |BM|$. The plane MAA' meets the line B'C' at E. Show that $|B'E| = m$. The plane MDB' meets the line A'D' at F. Show that $|D'F| = m$. Hence or otherwise show how to construct the point P at the intersection of L and the plane A'B'C'D'. Find the distance between P and the line A'B' and the distance between P and the line A'D' in terms of m. Find a relation between these two distances that does not depend on m. Find the locus of M. Let S be the envelope of the line L as M varies. Find the intersection of S with the faces of the cube.

## 11th Vietnam 1972 problems

**Problem A1**
Let $x = \cos \alpha$, $y = \cos n\alpha$, where n is a positive integer. Show that for each x in the range [-1, 1], there is only one corresponding y. So consider y as a function of x and put $y = T_n(x)$. Find $T_1(x)$ and $T_2(x)$ and show that $T_{n+1}(x) = 2x\, T_n(x) - T_{n-1}(x)$. Show that $T_n(x)$ is a polynomial of degree n with n roots in [-1, 1].

**Problem A2**
For any positive integer n, let $f(n) = \sum (-1)^{(d-1)/2}$ where the sum is taken over all odd d dividing n. Show that:

$f(2^n) = 1$
f(p) = 2 for p a prime congruent to 1 mod 4
f(p) = 0 for p a prime congruent to 3 mod 4
$f(p^n) = n+1$ for p a prime congruent to 1 mod 4
$f(p^n) = 1$ for p a prime congruent to 3 mod 4, and n even
$f(p^n) = 0$ for p a prime congruent to 3 mod 4, and n odd

Show that f(mn) = f(m) f(n) for m and n relatively prime. Find $f(5^4 11^{28} 17^{19})$ and f(1980). Show how to calculate f(n).

**Problem B1**
ABC is a triangle. U is a point on the line BC. I is the midpoint of BC. The line through C parallel to AI meets the line AU at E. The line through E parallel to BC meets the line AB at F. The line through E parallel to AB meets the line BC at H. The line through H parallel to AU meets the line AB at K. The lines HK and FG meet at T. V is the point on the line AU such that A is the midpoint of UV. Show that V, T and I are collinear. [Next part unclear.]
**B2.** ABCD is a regular tetrahedron with side a. Take E, E' on the edge AB such that AE = a/6, AE' = 5a/6. Take F, F' on the edge AC such that AF = a/4, AF' = 3a/4. Take G, G' on the edge AD such that AG = a/3, AG' = 2a/3. Find the intersection of the planes BCD, EFG and E'F'G' and its position in the triangle BCD. Calculate the volume of EFGE'F'G' and find the angles between the lines AB, AC, AD and the plane EFG.

## 12th VMO 1974

**Problem A1**
Find all positive integers n and b with $0 < b < 10$ such that if $a_n$ is the positive integer with n digits, all of them 1, then $a_{2n} - b\, a_n$ is a square.

**Answer**
b = 2 works for any n
b = 7 works for n = 1

**Solution**
$a_n = (10^n-1)/9$, so we need $10^{2n} - b10^n + (b-1)$ to be a square. For b = 2, this is true for all n. Note that $(10^n - c)^2 = 10^{2n} - 2c10^n + c^2$, so if b = 2c, we need $c^2 = b-1 = 2c - 1$ and hence c = 1. So b = 4, 6, 8 do not work. Equally $10^{2n} > 10^{2n} - 10^n > (10^n - 1)^2 = 10^{2n} - 2\cdot 10^n + 1$, so it cannot be a square for b = 1. Similarly, $(10^n - 1)^2 = 10^{2n} - 2\cdot 10^n + 1 > 10^{2n} - 3\cdot 10^n + 2 > 10^{2n} - 4\cdot 10^n + 4 = (10^n - 2)^2$, so b = 3 does not work. Similarly, $(10^n - 2)^2 = 10^{2n} - 4\cdot 10^n + 4 > 10^{2n} - 5\cdot 10^n + 4 > 10^{2n} - 6\cdot 10^n + 9 = (10^n - 3)^2$, so b = 5 does not work. Similarly, $(10^n - 3)^2 = 10^{2n} - 6\cdot 10^n + 9 > 10^{2n} - 7\cdot 10^n + 6 > 10^{2n} - 8\cdot 10^n + 16 = (10^n - 4)^2$ for n > 1, so b = 7 does not work, except possibly for n = 1. Since $11 - 7 = 2^2$, b = 1 does work for n = 1. Finally, $(10^n - 4)^2 = 10^{2n} - 8\cdot 10^n + 16 > 10^{2n} - 9\cdot 10^n + 8 > 10^{2n} - 10\cdot 10^n + 25 = (10^n - 4)^2$ for n > 1, so b = 9 does not work, except possibly for n = 1. It is easy to check it does not work for n = 1.

**Problem A2**
(1) How many positive integers n are such that n is divisible by 8 and n+1 is divisible by 25?
(2) How many positive integers n are such that n is divisible by 21 and n+1 is divisible by 165?
(3) Find all integers n such that n is divisible by 9, n+1 is divisible by 25 and n+2 is divisible by 4.

**Answer**
infinitely many, none, n = 774 mod 900

**Solution**
(1) We need n = 0 mod 8 and -1 mod 25. Hence n = 24 mod 200.
(2) We need n = 0 mod 21, n = -1 mod 165. But 3 divides 165, so we require n = 0 mod 3 and 2 mod 3, which is impossible.
(3) We need n = 0 mod 9, -1 mod 25, 2 mod 4. Hence n = 99 mod 225, and 2 mod 4, so n = 774 mod 900

**Problem B1**
ABC is a triangle. AH is the altitude. P, Q are the feet of the perpendiculars from P to AB, AC respectively. M is a variable point on PQ. The line through M perpendicular to MH meets the lines AB, AC at R, S respectively. Show that ARHS is cyclic. If M' is another position of M with corresponding points R', S', show that the ratio RR'/SS' is constant. Find the conditions on ABC such that if M moves at constant speed along PQ, then the speeds of R along AB and S along AC are the same. The point K on the line HM is on the other side of M to H and satisfies KM = HM. The line through K perpendicular to PQ meets the line RS at D. Show that if $\angle A = 90^{\circ}$, then $\angle BHR = \angle DHR$.

**Problem B2**
C is a cube side 1. The 12 lines containing the sides of the cube meet at plane p in 12 points. What can you say about the 12 points?