

## Aufgaben zu partiellen Ableitungen

### Aufgabe

Ermitteln Sie die unbestimmten Integrale mit Hilfe der Methode der partiellen Integration.

1)  $\int x \cdot e^x dx$

2)  $\int \ln x dx$

3)  $\int \ln^2 x dx$

4)  $\int x^2 \sin x dx$

5)  $\int \sin^2 x dx$

6)  $\int e^x \sin x dx$

7)  $\int \frac{\operatorname{arsinh} x}{x^2} dx$

8)  $\int x \ln x dx$

9)  $\int \frac{\ln x}{x} dx$

## Lösungen

1)

$$\int x \cdot e^x dx = x \cdot \int e^x - \int \left(1 \cdot \int (e^x dx)\right) dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + c = e^x(x-1) + c$$

2)

$$\int \ln x dx = \int (\ln x \cdot 1) dx = \ln x \cdot x - \int \left(\frac{1}{x} \cdot x\right) dx = x \ln x - \int 1 dx = x \ln x - x + c$$

3)

$$\begin{aligned} \int \ln^2 x \cdot x dx &= \int \ln x \cdot \ln x dx = \ln x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx = x \ln^2 x - x \ln x - \int (\ln x - 1) dx = \\ &= x \ln^2 x - x \ln x - (x \ln x - x) + x + c = x \ln^2 x - x \ln x - x \ln x + x + x + c = x \ln^2 x - 2x \ln x + 2x + c \end{aligned}$$

4)

$$\begin{aligned} \int x^2 \sin x dx &= x^2(-\cos x) - \int 2x \cdot (-\cos x) dx = -x^2 \cos x + 2 \int -x \cos x dx = \\ &= -x^2 \cos x + 2(x \sin x - \int 1 \cdot \sin x dx) = -x^2 \cos x + 2(x \sin x + \cos x) + c = \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + c \end{aligned}$$

5)

$$\begin{aligned} \int \sin^2 x dx &= \int \sin^2 x \cdot 1 dx = \sin^2 x \cdot x - \int 2 \sin x \cos x \cdot x dx = x \sin^2 x - \int x \sin 2x dx = \\ &= x \sin^2 x - \left(x \int \sin 2x dx - \int 1 \cdot \left(\int \sin 2x dx\right) dx\right) = x \sin^2 x - x \frac{-\cos 2x}{2} + \int \frac{-\cos 2x}{2} dx = \\ &= x \sin^2 x + \frac{x \cos 2x}{2} - \frac{1}{2} \int \cos 2x dx = x \sin^2 x + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} + c = \\ &= \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c = \frac{x \cos^2 x}{2} + \frac{x \sin^2 x}{2} - \frac{\sin 2x}{4} + c = \\ &= \frac{x}{2}(\cos^2 x + \sin^2 x) - \frac{\sin 2x}{4} + c = \frac{x}{2} - \frac{\sin 2x}{4} + c \end{aligned}$$

6)

$$\begin{aligned} \int e^x \sin x dx &= e^x \sin x - \int e^x \cos x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx \\ \int e^x \sin x dx &= \frac{1}{2} e^x (\sin x - \cos x) + c \end{aligned}$$

7)

$$\int \frac{1}{x^2} \operatorname{arsinh} x dx = -\frac{1}{x} \operatorname{arsinh} x + \frac{dx}{x\sqrt{1+x^2}} = -\frac{1}{x} \operatorname{arsinh} x + \int \frac{dz}{z^2-1} = -\frac{1}{x} \operatorname{arsinh} x - \operatorname{arcoth} \sqrt{x^2+1} + c$$

8)

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c$$

9)

$$\int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x + c$$